Counting :- ① Unit - 5 :

Sum Rule :- If a first job can be done in n_1 ways and a second job in n_2 ways and if these jobs can't be done at same time then there are (n_1+n_2) ways to do either job.

Exp :- ① If a computer project can be choosen by a student from one of Four lists. contain 21, 19, 17 & 15 possible projects. How many possible projects are there to chose from.

Ans :- Possible Projects = 21+19+17+15

Exp :- ② :-

14 boys & 12 girls in class find a number of ways for selecting one CR.

Total ways = 14+12 = 26

4,10 16 20 32 42,44,50

1,12, 6,12, 13, 17, 29, 37, 38, 51, 54, 56, 58 Date : 22/10/12

~~22~~, ~~16~~, 40, ~~56~~, ~~26~~, ~~1~~, ~~44~~, 9, ~~42~~

Product Rule:- Let a Procedure can be broken into two jobs. If first job can be done in n_1 ways & ~~second job can~~ n_2 ways ~~then there are~~ to do second job after first job has been then there are $n_1 \cdot n_2$ ways to do the procedure.

Exp ① Three person, enter in five seated car How many ways they can their seats.

ans:- $\overset{I}{5} \times \overset{II}{4} \times \overset{III}{3} = 60$

Exp:- for a set of 6 true/False question, find the no of ways of answering all question.

ans:- Total number $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 64$

Exp:- ② How many number plates from 2 letters followed by four digit.

ans:- $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 678{,}000$

Exp ③ Number Plates, from two letter followed by three digit (first digit should not zero)

Total $= 26 \cdot 26 \cdot 9 \times 10 \times 10$
$= 608400$

Permutation: An order selection of r objects from a set of n objects is called permutation & rebetition is not allowed.

$$\boxed{P(n,r) = \frac{n!}{(n-r)!} = {}^{n}P_{r}}$$

Exp:- A, B, C, D, E, F. Find the number of "three letters words" using only the given six letters without repeation.

$$\text{Total words} = {}^{6}P_{3} = \frac{\lfloor 6}{\lfloor 6-3} = \frac{6\times5\times4\,\cancel{\lfloor 3}}{\cancel{\lfloor 3}}$$

$$= 120$$

Combination:- An unorder selection of r objects from a set of n objects is called combingation. If all the elements are distinct & repeation is not allowed the

$$\boxed{C(n,r) = \frac{n!}{r!\cdot(n-r)!} = {}^{n}C_{r}}$$

Exp:- How many committees of three can be formed from eight People.

$$\text{total} = {}^{8}C_{3} = \frac{\lfloor 8}{\lfloor 5 \times \lfloor 3} = \frac{8\times7\times\cancel{6}\times\cancel{\lfloor 5}}{\cancel{\lfloor 5}\times\cancel{\lfloor 3}}$$

$$= 56$$

Exp:- A farmer buys 3 Cow, 2 Pigs & 4 hens from 6 Cows, 5 Pigs & 8 hens. How many choice farmer have

ans:-
Total choice $= {}^6C_3 \cdot {}^5C_2 \cdot {}^8C_4$

$= \frac{6!}{3! \times 3!} \cdot \frac{5!}{2! \times 3!} \cdot \frac{8!}{4! \cdot 4!}$

$= \frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times \cancel{3!}} \times \frac{5 \times 4!}{2! \times 3!} \times \frac{8 \times 7 \times \cancel{6}^{2} \times 5 \times \cancel{4!}}{4! \times \cancel{4!}}$

$= 112$

$= \frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times \cancel{3!}} \times \frac{5 \times 4!}{\cancel{2! \times 3!}} \times \frac{8 \times 7 \times \cancel{6}^{4} \times 5 \times \cancel{4!}}{\cancel{4! \times 4!}}$

$= \underline{14000}$

Restricted Permutation:-

(i) k always include $P(n-k, r-k) \cdot P(r, k)$

(ii) k never included $P(n-k, r)$

Exp:- How many Three letter words from Beauty if
(i) B, U & T are not to be include.
(ii) ______ always include

(ii) $n = {}^3P_3 = 6$ (i) ${}^3P_0 \cdot {}^3P_3$
$= 1 \cdot 3!$
$= \underline{6}$

(i) ~~$n = {}^3P$~~

Restricted combination:

(i) Always included $^{n-k}C_{r-k}$

(ii) never includ $^{n-k}C_r$

Exp:- 25 People, 10 are selected for committee
Three member A, B, C will join all or none.

(i) A, B, C included
(ii) A, B ~~are~~ C are not inclu

(i) $n = {}^{22}C_7$

(ii) $n = {}^{22}C_{10}$

Permutation with repeatitions:-

The numbe of permutation of n objects of which n_1 ~~are, ... n_r~~ are, are alike, n_2 are alike ... a_r are alike

$$P(n_1, n_2, \ldots P_{n_r}) = \frac{\lfloor n}{\lfloor n_1 \cdot \lfloor n_2 \ldots \lfloor n_r}$$

Exp:- How many words can be formed using the all letters of "BENZENE"

$$n = \frac{7!}{\lfloor 3 \cdot \lfloor 2} = \frac{7 \times \overset{3}{\cancel{6}} \times 5 \times 4 \times \cancel{\lfloor 3}}{\cancel{\lfloor 3 \times \lfloor 2}}$$

$= 14 \cdot 30$

$= 420$

Q5 Find n if $2P(n,2)+50=P(2n,2)$

$n=5$

$$2\frac{\lfloor n}{\lfloor n-2}+50=\frac{\lfloor 2n}{\lfloor 2n-2}$$

$$2\times\frac{n(n-1)\times\cancel{\lfloor n-2}}{\cancel{\lfloor n-2}}+50=\frac{2n(2n-1)\cancel{\lfloor 2n-2}}{\cancel{\lfloor 2n-2}}$$

$$2n^2-2n+50=4n^2-2n$$

$$2n^2=50$$

$$n^2=25$$

$$n=\pm 5$$

n can't be negative so

$$\boxed{n=5}$$

Ans

(4)

Q2 11 friend

(a) how many way 5 of them $^{11}C_5$

(b) Two are married [both come or both not

$^{9}C_5 + {^{9}C_3}$

(c) Two will not come together

$= 2 \cdot {^{9}C_4}$

$^{9}C_4 + {^{9}C_4}$

(2) Li. 10 out of 13

(a) How many choice

(b) First two question

(c) First or second but not both $^{11}C_9 + {^{11}C_9}$

(d) There exactly first five $^{5}C_3 \cdot {^{8}C_7}$

(E) atleast Three out of first F

ans $^{5}C_3 \cdot {^{8}C_7} + {^{5}C_4} \cdot {^{8}C_6} + {^{5}C_5} \cdot {^{8}C_5}$

$\sum_{n=0}^{\infty} a_n z^n - 4\sum_{n=2}^{\infty} a_{n-1} z^n + 4\sum_{n=2}^{\infty} a_{n-2} z^n = A(z)$

$A(z) = \sum_{n=0}^{\infty} a_n z^n$

$0 = A(z) - [a_0 + a_1 z] - 4z[A(z) - a_0] + 4z^2 A(z)$

$A(z)[4z^2 - 4z + 1] + [\ldots] = 0$

$\frac{2z - 1}{1} = \frac{4z^2 - 4z + 1}{2z - 1}$

Pigeonhole Principle:-

If n pigeonholes are occupied by $n+1$ pigeons then atleast one Pigeon hole is occupied by more than 2 Pigeons.

Exp:- ① ~~27 student 26 alphabet.~~

Extended Pigeon hole Principle:-

① If n Pigeon holes are occupied by $kn+1$ or more pigeons then atleast one pigeon hole is occupied by $k+1$ more pigeons.

Exp:- ① Find the no of student three born in same month.

ans:- $k+1=3$, $k=2$

$n=12$

$\therefore$ minimum number of stud $= kn+1$

$= 2\times12+1$

$= 25$

Exp:- 2→ suppose Red, Blue & white socks. Find the minimum no of socks that two pair of same color.

$k+1=4$, $k=3$, $n=3$

min no $= kn+1 = 3\times3+1 = 10$

Recurrence Relation: (Difference Equation)

Recurrence relations are used to define the terms of a sequence.

A sequence is a function whose domain in some infinite set of integers ~~(often N) whose domain in some inf s~~ (often N) and whose range is a set of real numbers.

Exp:-

① Consider a sequence $2^0, 2^1, 2^2, \ldots$ — general term can be specified by the expression

$$\boxed{a_r = 2^r, r \geq 0}$$

or

$$\boxed{a_r = 2 \cdot a_{r-1}} \text{ and } \boxed{a_0 = 1}$$

Exp:- ② 1, 3, 5, 7, 9, 11, — — — by recurrence relⁿ

$$\boxed{a_n = a_{n-1} + 2} \text{ with } \boxed{a_0 = 1}$$

Exp ③:- fibonacci sequence by recurrence relⁿ

$$\boxed{a_n = a_{n-1} + a_{n-2}} \text{ with } \boxed{a_0 = 0 \text{ and } a_1 = 1}$$

Order of Recurrence Relation:

Difference betn the largest and the smallest subscript appearing in the relation.

Exp:-

$$a_n - 3a_{n-1} + 2a_{n-2} = 0$$

$$\text{order} = n-(n-2)$$
$$= 2$$

Degree of Recurrence Relation:

The degree of Recurrence Reln is defined to be the highest power of a_n.

Exp:-

$$\boxed{a_n^4 + 18\, a_{n-2}^3 + a_{n-3} = 0}$$

Degree $= 4$

Homogeneous & Non-homogeneous Recurrence Reln:

$$\boxed{c_0 a_r + c_1 a_{r-1} + c_2 a_{r-2} + \cdots c_k a_{r-k} = f(r)}$$

if $f(r) = 0$ then Homogenous

if $f(r) \neq 0$ then non Homogeneous.

if degree of left side is equal then linear.

Solution of Linear Recurrence Rel.^{n} with constant coefficients:

1. **Homogeneous solution:-**

Make characteristics equation and find root.

(a) If the roots are distinct say m_1, m_2, m_3 --- then homogeneous solution is

$$a_r^h = C_1 (m_1)^r + C_2 (m_2)^r + C_3 (m_3)^r + \cdots C_n (m_n)^r$$

(b) If the two roots are repeated say $m_1 = m_2$

then $$a_r^h = (C_1 + r C_2)(m_1)^r$$

Exp:- $a_r - 6a_{r-1} + 9a_{r-2} = 0$

Char^r equ^n:

$$\alpha^2 - 6\alpha + 9 = 0$$

$(a-b)^3$

$$(\alpha - 3)^2 = 0$$

$$\alpha = 3, 3$$

$(x-3)(x-3)(x-3)$

$x^3 - 27 - 9x^2 + 27x$

$$\boxed{a_r^h = (c_1 + r c_2)\, 3^r}$$

Exp ② $\boxed{a_r - 6a_{r-1} + 8a_{r-2} = 0}$

Char^r equation

$$\alpha^2 - 6\alpha + 8 = 0$$

$$\alpha^2 - 4\alpha - 2\alpha + 8 = 0$$

$$\alpha(\alpha - 4) - 2(\alpha - 4) = 0$$

$$(\alpha - 2)(\alpha - 4) = 0$$

$$\alpha = 2, 4$$

$$\boxed{a_r^h = c_1\, 2^r + c_2\, 4^r}$$

Exp:-③ $a_r - 5a_{r-1} + 6a_{r-2} = 0$

$a_0 = 2$ & $a_1 = 5$

Char^r eq

$$\alpha^2 - 5\alpha + 6 = 0$$

$$\alpha^2 - 3\alpha - 2\alpha + 6 = 0$$

$$\alpha(\alpha - 3) - 2(\alpha - 3) = 0$$

$$\alpha = 2, 3$$

$$\boxed{a_r^{h} = c_1 2^r + c_2 3^r} \quad (3)$$

$r = 0$.

$a_0 = c_1 + c_2 = 2$ ——— (1)

$r = 1$

$a_1 = 2c_1 + 3c_2 = 5$ ——— (2)

$2c_1 + 2c_2 = 4$

$c_2 = 1$

$c_1 = 1$

$$\therefore \boxed{a_r^{h} = 2^r + 3^r}$$

(2) Particular solution:-

Case (i):- If $f(r)$ in the form of

$F_1 r^t + F_2 r^{t-1} + F_3 r^{t-2} + \text{---} \; F_t r + F_{t+1} r^0$

then Particular solution is

$a_r^{p} = P_1 r^t + P_2 r^{t-1} + \text{--} \; P_t r + P_{t+1}$

Exp:- $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$

$\therefore$ Char^r eqn

$\alpha^2 + 5\alpha + 6 = 0$ after solving

$\alpha = -3,\ \alpha = -2$

$$\boxed{a_r^h = C_1(-3)^r + C_2(-2)^r}$$

Particular solution

$a_r^p = P_1 r^2 + P_2 r + P_3$

$\downarrow$

$(P_1 r^2 + P_2 r + P_3) + 5[P_1(r-1)^2 + P_2(r-1) + P_3]$

$+ 6[P_1(r-2)^2 + P_2(r-2) + P_3] = 3r^2 - 2r + 1$

$\Downarrow$

$r^2[P_1 + 5P_1 + 6P_1] + r[P_2 - 10P_1 + 5P_2 - 24P_1 + 6P_2]$

$+ P_3 + 5P_1 - 5P_2 + 5P_3 + 24P_1 - 12P_2 + 6P_3$

$= 3r^2 - 2r + 1$

$r^2(12P_1) + r[-34P_1 + 12P_2] + 29P_1 - 17P_2 + 12P_3$
$= 3r^2 - 2r + 1$

(4)

$12P_1 = 3$

$P_1 = \frac{1}{4}$

$-34P_1 + 12P_2 = -2$

$-34 \times \frac{1}{4} + 12P_2 = -2$

$P_2 = \frac{13}{24}$

$29P_1 - 17P_2 + 12P_3 = 1$

$29 \times \frac{1}{4} - 17 \times \frac{13}{24} + 12P_3 = 1$

$P_3 = \frac{71}{288}$

$a_r^P = \frac{1}{4}r^2 + \frac{13}{24}r + \frac{71}{288}$

$\therefore \boxed{a_r^t = a_r^h + a_r^P}$

$a_r^T = C_1(-3)^r + C_2(-2)^r + \frac{1}{4}r^2 + \frac{13}{24}r + \frac{71}{288}$

Exp:- (2) $a_r - 5a_{r-1} + 6a_{r-2} = 1$

Characteristic equation

$\alpha^2 - 5\alpha + 6 = 0$

$\alpha - 3\alpha - 2\alpha + 6 = 0$

$\alpha(\alpha - 3) - 2(\alpha - 3) = 0$

$\alpha = 2, \alpha = 3$

$a_r^h = C_1(2)^r + C_2(3)^r$

Particular solⁿ

$a_r^p = P$

Put in

$P - 5P + 6P = 1$

$2P = 1$

$P = \frac{1}{2}$

$\therefore a_r^p = \frac{1}{2}$

$\therefore a_r^T = C_1(2)^r + C_2(3)^r + \frac{1}{2}$

Particular solution case (ii):-

if $f(r)$ is in the form of

$[F_1 r^t + F_2 r^{t-1} + \cdots F_t r + F_{t+1}] \cdot \beta^r$

where β is not charʳ Root of the equation then

$a_r^p = [P_1 r^t + P_2 r^{t-1} + \cdots P_t r + P_{t+1}] \cdot \beta^r$

Exp:- $a_r + a_{r-1} = 3r \cdot 2^r$

Charʳ equatiⁿ

$\alpha + 1 = 0$

$\alpha = -1$

$a_r^h = C_1(-1)^r$

5

$a_r' = (P_1 r + P_2)\cdot(2)^r$

$(P_1 r + P_2)(2)^r + [P_1(r-1) + P_2](2)^{r-1} = 3r\,2^r$

$[2P_1 r + 2P_2 - P_1]\,2^r = 3\cdot r\cdot 2^r$

$2P_1 = 3$

$P_1 r\cdot 2^r + P_2\cdot 2^r + \frac{P_1\, r\cdot 2^r}{2} - \frac{P_1}{2}\cdot 2^r + \frac{P_2}{2}\cdot 2^r = 3r\cdot 2^r$

$\left(P_1 + \frac{P_1}{2}\right)\cdot r\cdot 2^r + \left[P_2 + \frac{P_2}{2} - \frac{P_1}{2}\right]\cdot 2^r = 3r\,2^r$

$\frac{3P_1}{2} = 3 \quad \& \quad \frac{3P_2}{2} - \frac{P_1}{2} = 0$

$P_1 = 2 \qquad P_2 = \frac{2}{3}$

$$\boxed{a_r^T = C_1(-1)^r + \left(2r + \frac{2}{3}\right)2^r}$$

Particular solution: case (iii)

if $f(r)$ is of the form

$[F_1 r^t + F_2 r^{t-1} + F_3 r^{t-2} \cdots F_t r + F_{t+1}]\cdot\beta^r$

& β is root of Char^r equation then

$$a_r^P = r^m[P_1 r^t + P_2 r^{t-1} + \cdots P_t r + P_{t+1}] \cdot \beta^r$$

where m is repeated no of root (β).

<u>Exp:- ①</u> $a_r - 2a_{r-1} = 3 \cdot 2^r$ ——— ①

Char^r equation

$$\alpha - 2 = 0$$

$$\alpha = 2$$

$$\boxed{a_r^h = c_1 (2)^r}$$

Particular sol^n

$$a_r = r^1 [P] \, 2^r$$

$a_r = r P 2^r$ Put in equ (1)

$$rP2^r - 2(r-1)P2^{r-1} = 3 \cdot 2^r$$

$$rP2^r - (r-1)P2^r = 3 \cdot 2^r$$

$$\cancel{rP2^r} - \cancel{rP2^r} + P2^r = 3 \cdot 2^r$$

$$P = 3$$

$$\boxed{a_r^T = c_1 2^r + 3r2^r}$$

$(9-5)^2 = 64$

$729 - 125 - 3 \cdot 81 \cdot 5 + 3 \cdot 9 \cdot 25$

$729 - 125 - 1215 + 675$

$(r^2+1-2r)(r-1)$

$r^3 + r - 2r^2 - r^2 - 1 + 2r$

$r^3 - 3r^2 - 1 + 3r$

6

Exp:- $a_r - 6a_{r-1} + 9a_{r-2} = r \cdot 3^r$ —— (1)

Characteristics equation:-

$\alpha^2 - 6\alpha + 9 = 0$

$\alpha = 3, 3$

$a_r^H = (C_1 + rC_2)(3)^r$

Particular solution:-

$a_r^P = r^2[P_1 r + P_2] \cdot 3^r$ Put in equ (1)

$\Rightarrow r^2[P_1 r + P_2]3^r - 6(r-1)^2[P_1(r-1) + P_2]3^{r-1}$

$+ 9(r-2)^2[P_1(r-2) + P_2]3^{r-2} = r \cdot 3^r$

$r^2[P_1 r + P_2]\cancel{3^r} - 2(r-1)^2[P_1(r-1) + P_2]\cancel{3^r}$

$+ 3(r-2)^2[P_1(r-2) + P_2]\cancel{3^r} = r \cdot \cancel{3^r}$

$\Rightarrow r^2[P_1 r + P_2] - 2(r^2 + 1 - 2r)[P_1(r-1) + P_2]$ $P_1 r - P_1$

$+ (r^2 + 4 - 4r)[P_1(r-2) + P_2] = r$

$\Rightarrow r^3[P_1 - 2P_1 + P_1] + r^2[P_2 + 2P_1 - 2P_2 + 4P_1$

$- 2P_1 + P_2 - 4P_1$

$+ 4P_1$ $- 4P_2$

$\cancel{P_1 r^3 + P_2 r^2}$

$$P_1 r^3 + P_2 r^2 - 2\left[P_1 (r-1)^3 + P_2 (r-1)^2\right]$$

$$+ P_1 \left[(r-2)^3 + P_2 (r-2)^2 = r\right.$$

$$P_1 r^3 + P_2 r^2 - 2\left(P_1 (r^3 - 1 - 3r^2 + 3r) + P_2 (r^2 + 1 - 2r)\right)$$

$$+ P_1 (r^3 - 8 - 6r^2 + 12r) + P_2 (r^2 + 4 - 4r) = r$$

$$\Rightarrow \cancel{P_1 r^3} + \cancel{P_2 r^2} - \cancel{2P_1 r^3} + \cancel{2P_1} + \cancel{6P_1 r^2} - \cancel{6P_1 r}$$

$$- \cancel{2P_2 r^2} - \cancel{2P_2} + \cancel{4P_2 r} + \cancel{P_1 r^3} - \cancel{8P_1} - \cancel{6P_1 r^2}$$

$$+ \cancel{12P_1 r} + \cancel{P_2 r^2} + \cancel{P_2 4} - \cancel{4P_2 r} = r$$

$$6P_1 r - 6P_1 + 2P_2 = r$$

Exp:- ① $a_r - a_{r-1} = 4$

Exp:- ② $a_r = 3a_{r-1} + 2r$

$a - 3a_{r-1} = 2r$

$$6P_1 = 1$$

$$P_1 = \frac{1}{6}$$

$$-6P_1 + 2P_2 = 0$$

$$P_2 = 3P_1$$

$$= 3 \times \frac{1}{6} = \frac{1}{2}$$

$$\boxed{a_r^T = (C_1 + rC_2)3^r + r^2\left[\frac{1}{6}r + \frac{1}{2}\right]3^r}$$

Generating Functions:

Generating function for infinite sequence $(a_0, a_1, a_2, \ldots)$ is the power series:

$$A(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \ldots$$

Exp:- ① The generating function of the sequence $\{1, 1, 1, \ldots 1, 1, 1 \ldots\}$

ans:- $A(z) = 1 + 1 \cdot z + 1 \cdot z^2 + 1 \cdot z^3 + \ldots$

$$A(z) = \frac{1}{1-z}$$

② The generating function for $(1, -1, 1, -1, 1, -1, \ldots)$

ans:- $A(z) = 1 - z + z^2 - z^3 + z^4 - z^5 + \ldots$

$$Az = \frac{1}{1+z}$$

③ Generating function for $(1, 2, 3, \ldots)$

$$A(z) = 1 + 2z + 3z^2 + 4z^3 \ldots \quad ①$$

multiply by z

$$z\,A(z) = z + 2z^2 + 3z^3 + \ldots \quad ②$$

$$(1-z)\,A(z) = 1 + z + z^2 + z^3 + \ldots$$

$$(1-z)\,A(z) = \frac{1}{1-z}$$

$$A(z) = \frac{1}{(1-z)^2}$$

(4) $C(8,0), C(8,1), C(8,2), \ldots C(8,8), 0, 0 \ldots$

ans:

$$A(z) = C(8,0) + C(8,1)z + C(8,2)z^2 + \ldots C(8,8)z^8$$

$$= \underline{(1+z)^8}$$

(5) Find G.F for sequence whose n^{th} term $a_n = n$.

$$\therefore a_0 = 0, a_1 = 1, a_2 = 2 \ldots$$

$$A(z) = 0 + 1z + 2z^2 + 3z^3 + 4z^4 \ldots$$

$$z\,A(z) = z^2 + 2z^3 + 3z^4 + \ldots$$

$$(1-z)\,A(z) = z + z^2 + z^3 + \ldots$$

$$\boxed{A(z) = \frac{z}{(1-z)^2}}$$

(6) Find the G.F for numeric function

$$a_n = \frac{1}{(n+1)!}$$

$$a_0 = \frac{1}{1!},\ a_1 = \frac{1}{2!},\ a_2 = \frac{1}{3!} \ldots$$

$$A(z) = 1 + \frac{1}{2!}z + \frac{1}{3!}z^2 + \ldots$$

$$= \frac{1}{z}\left[z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots\right]$$

$$= \frac{1}{z}\left[-1 + 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots\right] = \frac{e^z - 1}{z}$$

(8)

Exp:- Determine the numeric function for given G.F.

Note:-

$(1-x)^{-1}$

$1+x+x^2+x^3+x^4 ---$

(a) $A(z) = \frac{9}{1-5z}$

$= 9(1-5z)^{-1}$

$= 9(1+5z+(5z)^2+---$

$= 9\sum_{n=0}^{\infty} 5^n z^n$

$= 9 \cdot 5^n \quad , n \geq 0$

(b) $A(z) = \frac{10}{1-z} + \frac{12}{2-z}$

$= 10(1-z)^{-1} + 12(2-z)^{-1}$

$= 10(1+z+z^2+z^3+---) + \frac{12}{2}\left(1-\frac{z}{2}\right)^{-1}$

$= 10\sum_{n=0}^{\infty} 1^n z^n + 6 \left(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+---\right)$

$= 10\sum_{n=0}^{\infty} 1^n z^n + 6\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$

$= 10 \cdot 1^n + 6\left(\frac{1}{2}\right)^n \quad , n \geq 0$

(c) $A(z) = (1+z)^{m} + (1-z)^{m}$

$= {}^mC_0 + {}^mC_1 z + {}^mC_2 z^2 + \cdots$

$+ {}^mC_0 + {}^mC_1 (-z) + {}^mC_2 (-z)^2 + \cdots$

$a_n = \sum_{n=0}^{\infty} {}^mC_n z^n + \sum_{n=0}^{\infty} {}^mC_n (-z)^n$

$a_n = {}^mC_n + (-1)^n \, {}^mC_n$

$= {}^mC_n \left[1 + (-1)^n\right], \quad n \geq 0$

Note:-

(i) $S^{i}\, a_r = \begin{cases} 0 & 0 \le r \le i-1 \\ a_{r-i} & r \ge i \end{cases}$

(ii) $S^{-i}\, a_r = \{ a_{r+i} \quad r \ge 0 \}$

Exp:- (1) $a_r = \begin{cases} 1 & 0 \le r \le 10 \\ 2 & r \ge 11 \end{cases}$

find $S^{5} a_r$.

$$S^{5} a_r = \begin{cases} 0 & 0 \le r \le 4 \\ a_{r-5} & r \ge 5 \end{cases}$$

	a_0	1
$r=5$		
$r=6$	a_1	1
$\vdots$	$\vdots$	$\vdots$
	a_{10}	1
$r=15$		2
	a_{11}	
$r=16$		

$$\therefore S^{5} a_r = \begin{cases} 0 & 0 \le r \le 4 \\ 1 & 5 \le r \le 15 \\ 2 & r \ge 16 \end{cases}$$

Exp:- 2

$$a_r = \begin{cases} 1 & 0 \le r \le 20 \\ 2 & r \ge 21 \end{cases}$$

find $S^{11} a_r$ and $S^{-11} a_r$

$$S^{11} a_r = \begin{cases} 0 & 0 \le r \le 10 \\ a_{r-11} & r \ge 11 \end{cases}$$

	a_0	1
$r = 11$	a_1	1
$r = 12$	$\vdots$	1
$\vdots$	a_{20}	1
$r = 31$		2
$r = 32$	a_{21}	

$$\therefore S^{11} = \begin{cases} 0 & 0 \le r \le 10 \\ 1 & 11 \le r \le 31 \\ 2 & r \ge 32 \end{cases}$$

(ii) $S^{-11} a_r = \{ a_{r+11} \quad r \ge 0 \}$

	a_{11}	1
$r = 0$	a_{12}	1
$r = 1$	— —	1
$r = 9$	a_{20}	2
$r = 10$	a_{21}	

$$\therefore S^{-11} a_r \begin{cases} 1 & 0 \le r \le 9 \\ 2 & r \ge 10 \end{cases}$$

(11)

$$A(z) = (1-z)^{-1} + (1-2z)^{-1}$$

$$= 1+z+z^2+\cdots \quad 1+2z+(2z)^2+\cdots$$

$$= \sum_{r=0}^{\infty} 1^r z^r + \sum_{r=0}^{\infty} 2^r z^r$$

$$\boxed{a_r = 1+2^r}$$

<u>Exp:-2</u> $a_r - 2a_{r-1} + a_{r-2} = 2^r \quad , r \geq 2, a_0 = 2, a_1 = 1$

$$\sum_{r=2}^{\infty} a_r z^r - 2\sum_{r=2}^{\infty} a_{r-1} z^r + \sum_{r=2}^{\infty} a_{r-2} z^r \quad \sum_{r=2}^{\infty} 2^r z^r$$

$\cancel{A(z)(a_0 - a_1 z)}$

$$A(z) - a_0 - a_1 z - 2z\left[A(z) - a_0\right] + z^2 \cdot A(z)$$

$$= \frac{4z^2}{1-2z}$$

$$A(z)\left(z^2 - 2z + 1\right) - 2 - z + 4z = \frac{4z^2}{1-2z}$$

$$A(z)\,(1-z)^2 + (3z-2) = \frac{4z^2}{1-2z}$$

$$A(z) = \frac{4z^2}{(1-z)^2(1-2z)} + \frac{(2-3z)}{(1-z)^2}$$

$$\frac{A}{(1-z)} + \frac{B}{(1-z)^2} + \frac{C}{1-2z} + \left(\frac{A}{1-z} + \frac{B}{(1-z)^2}\right.$$

$$\frac{A(1-z)(1-2z)+B(1-2z)+C(1-z)^2}{(1-2z)(1-z)^2} \qquad \frac{A(1-z)+B}{(1-z)^2}$$

$$\frac{A[1-3z+2z^2]+B-2Bz+C+Cz^2-2Cz}{(1-2z)(1-z)^2} \qquad A-Az+B$$

$2A+C=4$

$-3A-2B-2C=0$

$A+2B+C=0$

$A+B=2$

$A=3$

$B=1$

$A+B=4$

$-A=0$

$B=-4$

$C=C$

Exp-3 :-

$a_r-5a_{r-1}+6a_{r-2}=2, \ r\geq 2$

$a_0=1, a_1=1$

ans :- $a_r=1$

$$=\frac{-4}{(1-z)^2}+\frac{4}{1-2z} + \frac{3}{(1-z)} + \frac{1}{(1-z)^2}$$

$$=\frac{-5}{(1-z)^2}+\frac{4}{(1-2z)}+\frac{3}{1-z}$$

$$=-5(1-z)^{-1}\cdot(1-z^{-1})+4(1-2z)^{-1}+3(1-z)^{-1}$$

$$=4(1+2z+(2z)^2+(2z)^3+\cdots)-3[1+z+z^2+z^3+\cdots]$$

$$-5[(1+z+z^2+z^3+\cdots)(1+z+z^2+z^3+\cdots)]$$

$$=4\sum_{r=0}^{\infty}2^r\cdot z^r+3\sum_{r=0}^{\infty}1^r\cdot z^r-5[1+2z+3z^2+\cdots]$$

$$-5\sum_{r=0}^{\infty}(r+1)\cdot z^r$$

$$a_r=4\cdot 2^r+3-5(r+1)$$

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Solution of Recurrence Relⁿ using Generating funⁿ:-

1

Exp:- $a_r - 3a_{r-1} + 2a_{r-2} = 0$, $r \geq 2$, $a_0 = 2$, $a_1 = 3$

Let $A(Z)$ is Generating funⁿ of sequence (a_n)

$$A(Z) = \sum_{r=0}^{\infty} a_r Z^r$$

multiply (1) by Z^r

$$a_r Z^r - 3a_{r-1} Z^r + 2a_{r-2} Z^r = 0$$

summing from $r=2$ to ∞ we have

$$\sum_{r=2}^{\infty} a_r Z^r - 3\sum_{r=2}^{\infty} a_{r-1} Z^r + 2\sum_{r=2}^{\infty} a_{r-2} Z^r = 0$$

$$[A(Z) - a_0 - a_1 Z] - \frac{3Z}{\cancel{2}}[A(Z) - a_0] + \frac{2Z^2}{\cancel{2}}[A(Z)] = 0$$

~~$a_0 = 2$, $a_1 = 3$~~

$$[A(Z) - 2 - 3Z] - \frac{3}{Z}[A(Z) - 2] + \frac{2}{Z^2}(A(Z)) = 0$$

$$Z^2 A(Z) - 2Z^2 - 3Z^3 - 3Z A(Z) + 6Z + 2A(Z) = 0$$

$$A(Z)[Z^2 - 3Z + 2] - 3Z^3 - 2Z^2 + 6Z = 0$$

$$A(z)\left[2z^2-3z+1\right]-a_0-a_1z+3za_0=0$$

$$a_0=2 \ \& \ a_1=3$$

$$A(z)\left(2z^2-2z-z+1\right)-2-3z+6z=0$$

$$A(z)\left(2z(z-1)-1(z-1)\right)=2-3z$$

$$A(z)=\frac{2-3z}{(1-z)(1-2z)}$$

$$=\frac{A}{1-z}+\frac{B}{1-2z}$$

$$=\frac{A-2Az+B-Bz}{(1-z)(1-2z)}.$$

$$-2A-B=-3$$

$$2A+B=3$$

$$A+B=2$$

$$A=1$$

$$B=1$$

$$A(z)=\frac{1}{1-z}+\frac{1}{1-2z}$$